Response of an ensemble of noisy neuron models to a single input

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Spike timing precision in response to a subthreshold stimulation can be enhanced by noise in ensembles of neurons [X. Pei, L. Wilkens, and F. Moss, Phys. Rev. Lett. **77**, 4679 (1996)]. We elucidate the mechanism underlying this phenomenon by computing the membrane potential distributions of ensembles of Hodgkin-Huxley neuron models. For small noise amplitudes, the membrane potential distribution takes on a Gaussian form centered on the resting potential, while for large fluctuations, there is a significant spread to lower potentials. These two regimes are separated by a relatively narrow band where the distributions transit rapidly from the Gaussian-like shapes to the spread ones. We argue that the optimal noise that maximizes the spike timing precision is situated close to this boundary. [S1063-651X(99)02911-6]

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I. INTRODUCTION

Trains of action potentials are the main carriers of information in nervous systems. Despite the large number of studies devoted to breaking the neural code, the detailed scheme of how neurons encode information into spike trains has not been completely elucidated yet. Mainly two hypotheses have been put forth. One, referred to as ''temporal coding,'' assumes that information is coded into the time of discharges. The other, ''rate coding,'' argues that information is encoded into window-averaged mean firing rates (for a discussion see [1]). The prevailing view has been that the unreliability of individual neurons together with interference by noise can alter spike timing so that temporal coding loses its efficiency in noisy conditions. The rate coding scheme is more robust to such perturbations since it averages out their effect [2].

By examining the response of an ensemble of Hodgkin-Huxley (HH) neuron models [3] to an excitatory postsynaptic potential (EPSP), Pei *et al.* [4] showed that, surprisingly, for subthreshold stimuli, the spike timing precision is maximal at an intermediate noise level. Their result suggests that temporal coding can in fact be robust to noise, and even be improved if the latter is tuned appropriately. The main purpose of the present study is to determine the essential mechanisms underlying this effect of noise.

Two factors determine the spike timing precision: (a) the distribution of the ''states'' of the units within the ensemble, (b) for a given state, the probability for the stimulus to evoke a discharge within some prescribed interval. The first factor accounts for the inhomogeneity of the ensemble. The second factor stems from the variability in the discharge times of the units even when they are in similar states of excitability. This variability is the result of the interplay between noise and the action potential generation mechanism. The focus of the present study is on the influence of the first factor on spike timing precision. The second factor, that is the discharge probability and the distribution of the latency (i.e., the time interval separating the onset of the input from the discharge) of an HH model and its reduced version have been investigated in Ref. [5].

In order to clarify the role of the ensemble inhomogeneity on the spike timing, we compute the distribution of the membrane potential of the units at the input arrival time. Starting from these distributions, we compute the response to stimuli of various size under two conditions: one in which the noise is maintained after the input arrival and the other in which the noise is suppressed. The second protocole eliminates the effect of noise on action potential generation, so that comparison between the two schemes unveils the influence of the first factor.

II. MEMBRANE POTENTIAL DISTRIBUTION

We estimated numerically the stationary distribution of the state variables (V,m,h,n) of the HH model from simulations (for description see the Appendix). We also introduced an extra variable t_{ref} that measures the time since its last discharge. The discharge is defined as an upward crossing of a spike-detection threshold $V_\theta = 50.0$ mV by the membrane potential, given that no such crossings occurred within 3 ms prior to it (i.e., $t_{\mathrm{ref}} > 3$ ms). Figure 1 shows how the distribution of the membrane potential V varied with the noise level. The membrane potential roughly reflects the level of excitability of the units (i.e., units with a larger membrane potential are in general more excitable than those with a smaller one). With this respect, units with $t_{\mathrm{ref}} < 3$ ms and $V > 6.0$ mV were not included in the distributions because they would not contribute to the response to an input. Furthermore, some of these units are in the process of the action potential so that they have large membrane potential values. Their inclusion in the distribution can thus give the impression that the ensemble is more excitable than it really is.

The curves in the upper panel of Fig. 1 represent the mean value (thick dashed curve) and the top and bottom limits of the distribution V_{99} (thick solid curves), that is, 99% of the distribution is below the upper thick solid lines and 99% is above the lower thick solid lines. Three regions can be distinguished. For low noise ($\sigma \leqslant 1.5$ μA/cm^2) the width of the distribution increases linearly. This regime corresponds to distributions that are similar to Gaussians (lower left panel in Fig. 1). As the noise is increased ($1.5 \leqslant \sigma \leqslant 2.5$ μA/cm^2), the lower thick solid line abruptly shifts to smaller values, indicating a marked widening of the membrane potential distribution at this point (lower middle panel

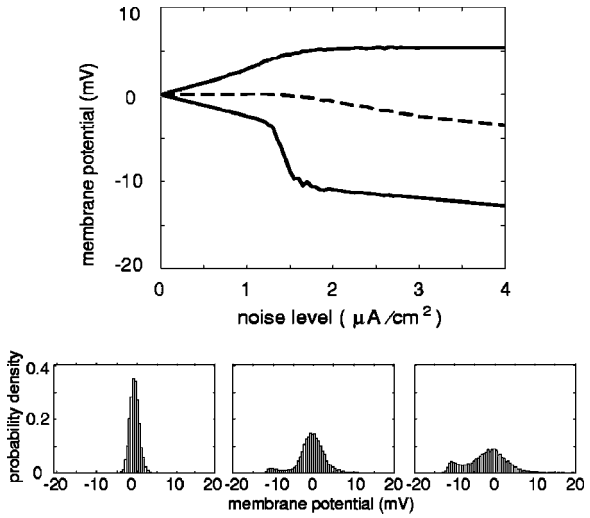

FIG. 1. Changes in the membrane potential distribution as the noise level is continuously increased for an ensemble of $N=10\,000$ HH units. Upper panel: Abscissa is noise intensity in ($\mu A/\mathrm{cm}^2$) and ordinate is voltage in (mV). The thick solid curves V_{99} show the top and bottom limits and the thick dashed curve shows the mean of the distribution. Units with $V>6.0$ mV or $t_{\mathrm{ref}}<3.0$ ms have been excluded. Three lower panels are the membrane potential distributions (containing all units) for $\sigma=1$, 2, and 3 $\mu\mathrm{A/cm}^2$ (from left to right). Abscissae: membrane potential in mV, ordinates, density in $(\mathrm{mV})^{-1}$.

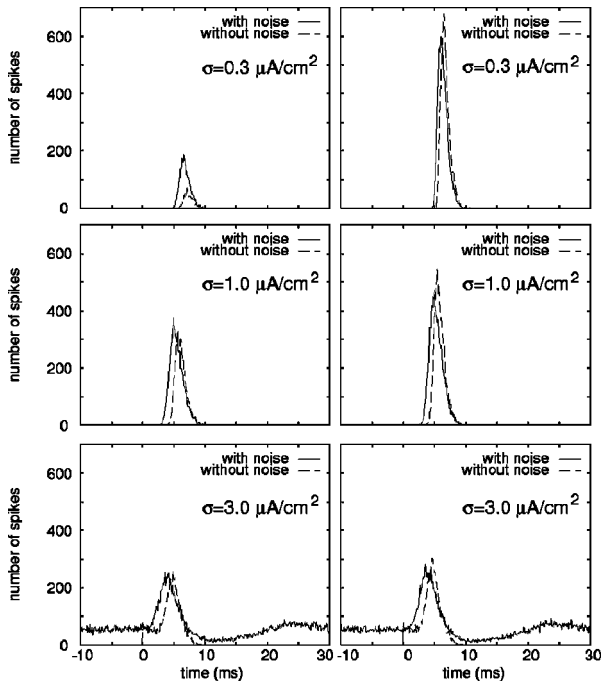

FIG. 2. PSTHs of stochastic HH ensembles (solid lines) and that of deterministic HH ensembles (dashed lines). Abscissa is time (ms) and ordinate is number of spikes (unitless). The input amplitudes are subthreshold $A=0.022$ $\mu\upsilon/\mathrm{cm}^2$ (left column) and suprathreshold 0.024 $\mu\upsilon/\mathrm{cm}^2$ (right column). The bin-size is 0.1 ms, and the number of units is $N=20\,000$.

in Fig. 1). This change is concurrent with the modification of the shape of the distribution itself, which is no longer close to a Gaussian. Finally at larger noise levels, the mean membrane potential steadily decreases, while the distribution spreads to lower values (lower right panel in Fig. 1) as attested by the lowest thick solid lines. The upper thick solid line is bounded by $V_\theta=6.0$ mV because of the exclusion of units with $V>6.0$ mV.

III. POST-STIMULUS TIME HISTOGRAM

We simulated the response of ensembles to a single EPSP and compute the corresponding post-stimulus time histogram (PSTH). Each simulation was run twice, once with and once without noise, starting from the same initial distribution corresponding to the stationary distribution at that particular noise level. Comparison of the response of ensembles in which noise is turned off at the stimulus onset (henceforth referred to as deterministic ensembles) with that of ensembles in which noise is maintained (henceforth referred to as stochastic ensembles) highlights the role of the initial state distribution. For deterministic ensembles, we refer to the noise intensity as that of the initial distribution.

Figure 2 represents examples of the PSTHs of deterministic and stochastic ensembles. The onset of the stimulation is taken as the time reference. The two columns in Fig. 2 illustrate the response of such ensembles to subthreshold (left column) and suprathreshold (right column) stimuli [6].

The PSTHs of both the deterministic (dashed lines) and the stochastic (solid lines) ensembles resemble those in Ref. [4]. At low noise levels, the subthreshold stimulus evokes a weak response (upper left panel). The peak height of this response increases with the noise (middle left panel) up to some optimal noise intensity, and then decays down (lower left panel) as the noise is further increased. For suprathreshold stimulation, increasing noise merely reduces progressively the peak height of the PSTH (the three panels in the right column of Fig. 2).

The remarkable point is the similarity between the response of the deterministic and the stochastic ensembles. In addition to displaying qualitatively the same dependence on the noise intensity, there is also quantitative agreement between the two models, in that the width and height of the PSTH are close, except for subthreshold stimulation at low noise levels. In this case, the stochastic ensemble produces a significantly larger response. Another noticeable difference is that the stochastic ensemble responds systematically faster than the deterministic one, with the difference between the response times (measured, for example, as the difference between the times at which the PSTHs reach their maxima) growing with the noise. Examination of the responses of individual units revealed that this phenomenon is due to the shortening of the latency by noise [5].

IV. SPIKE TIMING PRECISION

The propensity of the units to fire synchronously upon the arrival of the stimulation can be measured by the spike timing precision (STP) [4]. The STP, denoted by P is given by $P=P^m_{\mathrm{STH}}/W$, where P^m_{STH} is the maximum value of the five-point moving average of the PSTH and W is the width at P^m_{STH}/e.

As expected from the resemblance between the PSTHs of the deterministic and stochastic ensembles, their STPs also

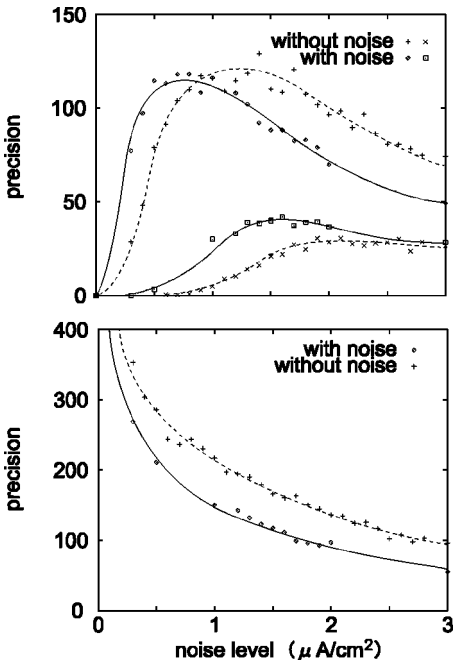

FIG. 3. Spike timing precision of an ensemble of HH units in response to subthreshold stimuli (upper panel, stimulus amplitude $A=0.016$ and 0.022 $\mu\upsilon/\mathrm{cm}^2$), and a suprathreshold stimulus (lower panel, stimulus amplitude $A=0.024$ $\mu\upsilon/\mathrm{cm}^2$). Abscissa is noise intensity ($\mu\mathrm{A}/\mathrm{cm}^2$) and ordinate is precision $[(\mathrm{ms})^{-1}]$. The number of units is $N=20\,000$ and the bin-size of the PSTH is 0.1 ms. The curves were provided for visual guidance.

display similar noise dependence. The panels in Fig. 3 show the noise intensity versus the STP for subthreshold (upper panel) and supratheshold (lower panel) stimulations for both deterministic and stochastic ensembles.

The effect of noise on the STPs depicted in these panels is consistent with the results in Ref. [4]. For subthreshold stimulation, the STP increases with the noise up to some optimal noise level, and then decays down as the noise is further increased. Conversely, the STP for suprathreshold stimulation monotonously decays with the noise. The comparison between the STPs of the deterministic and the stochastic ensembles confirms again the importance of the effect of noise on the initial distribution on the overall response of the ensembles. The differences between the STPs highlight the influence of the interplay between noise and action potential generation.

The notable difference between the STPs of the deterministic and stochastic models is that the former is shifted to larger noise levels in comparison with the latter. This holds for both subthreshold and suprathreshold stimulations. The main reason for this difference is that for subthreshold stimulation, the stochastic ensemble has a larger response at low noise levels (upper left panel in Fig. 2), so that the corresponding STP rises more rapidly. But at larger noise levels, the peak of the PSTH of the deterministic ensemble reaches similar heights, while its width remains narrower thereby leading to a larger STP for this system. For suprathreshold stimulation, mainly the latter phenomenon occurs, that is, the PSTH of the stochastic ensemble is wider than the deterministic one. This, combined with the fact that the two have comparable peak heights (that of the deterministic being slightly larger), yields a larger STP for the deterministic ensemble.

V. DISCUSSION

This study aimed to shed light onto the mechanisms underlying the noise-induced enhancement of STP [4] to subthreshold inputs and its progressive degradation for suprathreshold inputs in noisy ensembles of neuron models. The key observation that cutting the noise at the stimulus onset does not alter the response of the ensemble implies that the noise-induced changes in the membrane potential distribution play a central role in shaping the the response elicited by subthreshold as well as suprathreshold inputs. The following paragraphs clarify this role.

Schematically, the stationary states of ensembles can be divided into three regimes depending on the noise range. In the first regime (low noise levels) the membrane potential distribution is Gaussian-like. In the second regime (intermediate noise levels), the distribution abruptly departs from this shape by expanding towards lower membrane potential values. The third regime (large noise levels) follows this transition and is characaterized by a large proportion of hyperpolarized units, i.e., with low membrane potentials.

Remarkably, this classification also corresponded to the influence of noise on the STP. Indeed, for subthreshold stimuli, the STP increased in the first regime and decayed in the third one. The optimal noise level maximizing the STP corresponded to the transition between these regimes, corrected to take into account the extra variability due to the latency.

Phenomenologically, the relation between the membrane potential distribution and the STP can be understood in terms of the excitability of the ensemble. In the first regime, noise induced fluctuations bring some of the units closer to the firing threshold. These fluctuations induce little firing on their own. Thus overall, a fraction of the units within the ensemble is more excitable than at rest, while the spontaneous noise-induced firing remains low. As noise is increased within the range corresponding to the first regime, the spontaneous firing remains low while the excitability of the ensemble increases: larger fractions of units evolve closer to the firing threshold. This phenomenon, referred to as noise induced excitability, is responsible for the enhanced response in this range of noise intensities. As the noise is further increased beyond the first regime, the membrane potential fluctuations due to noise alone evoke discharges within the ensemble. Thus at the arrival time of the input a fraction of units within the ensemble are in the refractory stage recovering from noise-induced firing. These units do not respond to the input. Thus, noise-induced firing operates in antagonism with noise-induced excitability, tending to reduce the overall response of the ensemble. In the third regime when the noise is large, this factor becomes dominant and the response of the ensemble decreases. Thus, the maximum response is attained when the two antagonistic processes balance each other. In summary, we showed that the somewhat counterintuitive noise enhanced spike timing precision in response to subthreshold stimulation [4] can be understood in terms of the effect of noise on the membrane potential distribution and its implications on the overall excitability of the ensemble.

APPENDIX: THE HODGKIN-HUXLEY NEURON MODEL

The ensemble is a set of units in parallel. Units receive the same input and independent noise. All the spike trains are pooled at a summing center. The resulting train constitutes the output of the ensemble. The dynamics of the ith unit is determined by the following system of differential equations:

$$C_m \frac{dV_i}{dt} = g_{\mathrm{Na}} m_i^3 h_i (V_{\mathrm{Na}} - V_i) + g_{\mathrm{K}} n_i^4 (V_{\mathrm{K}} - V_i) + g_{\mathrm{L}} (V_{\mathrm{L}} - V_i) + g_{\mathrm{syn}}(t)(V_{\mathrm{syn}} - V_i) + \xi_i(t),$$

$$\frac{dm_i}{dt} = \alpha_m(V_i) - \gamma_m(V_i) m_i,$$

$$\frac{dh_i}{dt} = \alpha_h(V_i) - \gamma_h(V_i) h_i,$$

$$\frac{dn_i}{dt} = \alpha_n(V_i) - \gamma_n(V_i) n_i, \tag{A1}$$

where V_i, m_i, h_i, and n_i are the membrane potential, the activation and inactivation of the sodium current, and the activation of the potassium current. V_{Na}, V_{K}, and V_L are the reversal potentials of the sodium, potassium, and leak currents, and g_{Na}, g_K, and g_L are the corresponding maximal conductances. $\xi_i(t)$ represents white Gaussian noise satisfying $E(\xi_i(t))=0$ and $E(\xi_i(t)\xi_j(s))=\sigma^2\delta(t-s)$. V_{syn} is the synaptic current reversal potential and the synaptic conductance $g_{\mathrm{syn}}(t)$ is set to zero prior to the onset of the stimulus at time $t=0$ ms and varies as follows from then on [4]

$$g_{\mathrm{syn}}(t) = A \frac{t}{t_0} \exp\left[1 - \frac{t}{t_0}\right], \tag{A2}$$

where A is the maximum input conductance and $t_0 = 2.0$ ms is the characteristic time of the EPSP.

The auxiliary functions α_m, α_h, α_n, γ_m, γ_h, and γ_n are given in Ref. [3]. Parameter values are $C_m = 1.0$ μF/cm^2, $g_{\mathrm{Na}} = 120$ mυ/cm^2, $g_{\mathrm{K}} = 36$ mυ/cm^2, $g_L = 0.3$ mυ/cm^2, $\Phi = 1$, $V_{\mathrm{Na}} = V_{\mathrm{syn}} = 115$ mV, $V_{\mathrm{K}} = -12$ mV, and $V_L = 10.613$ mV. For these parameters the resting potential is at $V=0$ mV. Numerical simulations were carried out using the method in Ref. [7] adapted to take noise into account with a time step of 0.1 ms for the membrane potential distribution and 0.005 ms for the spike train. Controls run with smaller time steps and the Euler method yielded similar results.

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